Fractal Mathematics in Managed Care? How a Simple and Revealing Analysis Could Improve the Forecasting and Management of Medical Costs and Events

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In 1995, Christopher Barton, a research geologist who had worked for a decade with mathematician Benoît Mandelbrot, met with a group of U.S. research scientists interested in improving methods of forecasting hurricane wind speed at landfall and consequent damage. Noting that most hurricanes were small and of little consequence, while a few produced catastrophic damage, the group sought a more accurate way to forecast cataclysmic wind events. In response, Barton used U.S. historical data documenting maximum wind speeds for each hurricane at landfall dating from the year 1900 to create a cumulative frequency distribution (CFD) for wind speed, that is, the total number of hurricanes that had attained a given landfall wind speed for locations along the U.S. coast from Maine to Mexico. He then plotted the base 10 logarithms (log10) of CFD versus wind speed for each location. The resulting plot revealed a striking pattern of 2 separate mathematical functions. Noting that the wind speed where the 2 slopes intersected was approximately 40-50 meters per second (m/s), or about 90-110 miles per hour, Barton asked a group of research meteorologists at the National Oceanic and Atmospheric Administration's National Hurricane Research Center if there was any meteorological significance to a 40 m/s wind speed. The startled meteorologists replied that approximately 40 m/s signals the formation of the hurricane's eyewall and the transition from one physical process to another.1,2

In less than 15 years since that meeting, a growing number of forecasters and researchers—including meteorologists, geophysicists, and biologists—have applied similar mathematical approaches to the measurement and forecasting of a broad range of natural physical phenomena.3-7 When plotted on log-log scales, data representing the magnitude versus number of many natural phenomena often reveal much the same mathematical pattern as did the hurricane wind speeds plotted by Barton and his colleagues. Specifically, these data exhibit “power law” (fractal) scaling. In this editorial, we explain fractal scaling, highlight current health care trends that could make the use of fractal mathematical analysis increasingly important for the managed care industry, present a sample analysis from the physical sciences, and propose potential uses of the technique. We begin with 2 key concepts of fractal mathematics that apply across multiple venues and fields of study.

Key Concept 1: Self-Similarity Facilitates Event Forecasting. First, natural phenomena tend to form patterns in space and time that repeat over many orders of magnitude, a property that is described as “self-similarity.”7 An object in nature (space) displays self-similarity “if it can be decomposed into smaller copies of itself,”7 that is, if smaller components are essentially scaled-down versions of the larger object of which they are a part. Visible examples include some organs, such as lungs and the circulatory system, or cruciferous vegetables, such as broccoli or cauliflower. Events (time) display similar patterns of smaller and larger occurrences.

Mathematically, self-similarity is represented by a power law function rather than by the Gaussian statistics more familiar to many of us. Ordinary least squares linear regression assumes that for each 1 unit change in X (the independent variable), Y (the dependent variable) will increase or decrease by a fixed amount (the equation slope).8 In reality, though, most relationships between variables in nature are not identical across all values of a predictor variable; instead, they are similar, meaning that the direction of the relationship is the same, but the magnitude of the relationship differs at various values of the predictor, often by a great deal. Power law regression accounts for similarity by taking the multiplicative form $Y = MX^B$; where M and b are constants.7 The mathematical relationship is “scale invariant” or “scale independent,” meaning that the function is the same across a wide range of values of X and Y. For example, in a family of similar animals, body mass is the product of a power law function: mass = M × surface area$^{1/2}$, regardless of animal size.7 Fractal mathematics provides “a unified framework and explanation for many of these power laws.”7

The seminal character of self-similarity is profound and extends across multiple scientific disciplines. In his 1977 book introducing the concept of fractal mathematics, The Fractal Geometry of Nature, Mandelbrot observed that the science of geometry was often considered “cold” because of its “inability to describe” most objects that we observe in nature. “Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line,” he wrote. “Nature exhibits not simply a higher degree but an altogether different level of [geometric] complexity.”9 After considering the fractal patterns evident in numerous natural phenomena, including galaxies, coastlines, and air turbulence, Mandelbrot...
observed in retrospect that “exploring the consequences of self-similarity [proved] full of extraordinary surprises, helping me to understand the fabric of nature.” But much more than merely a vehicle to enhance appreciation of natural phenomena, the property of scale-independent (i.e., fractal) scaling has an important practical implication for event forecasting—when event CFDs are plotted against event magnitudes on a log-log scale, the scaling of small and frequent events facilitates the forecasting and understanding of larger, much less frequent, events.

**Key Concept 2: “Fractal Transitions” Provide Critical Insights.** Transformations of recalcitrant (typically skewed or curvilinear) data are not new to researchers familiar with quantitative techniques. As early as the 1970s, the SPSS (SPSS, Inc., Chicago IL) manual included advice on what statistician Marija Norusis later described as “coaxing” skewed and curvilinear distributions into linearity, using various methods that included logarithmic transformation of the data and the addition of polynomial terms to equations. More recently, many health care researchers routinely use general linear modeling, often employing non-Gaussian distributions, to characterize the relationships between various predictor variables and health care outcomes.

Much less well understood among health care researchers is the second, crucial concept derived from the growing understanding of fractal mathematics among physical scientists—the value of identifying the “fractal transition,” that is, the point at which the mathematical relationship between event CFD and magnitude changes from one power law to another power law. Specifically, when the cumulative numbers of physical phenomena are plotted against their magnitudes in log-log space, the point at which the slope changes—like the 40 m/s wind speed that signaled the eye of a hurricane in Barton’s study—often provides a fundamental insight into the scale at which the underlying processes of a phenomenon change. A familiar example from chemistry is a phase change (e.g., from water to steam or ice). Scholz (1995) observed that “many [geological] objects may exhibit self-similarity over only a limited range of scales and different characteristics at widely separated scales.” The roughness of “natural geological surfaces,” Scholz observed, exhibits such transitions, which “occur at wavelengths corresponding to characteristic lengths in the system that generates the surface and these lengths may provide clues to the underlying physics.” Thus, however tempting it might be to assume that “one power law fits all” in characterizing the relationship between the frequency and magnitude of events, doing so may cause us to miss an opportunity to learn about the processes that generate events, whether minor or catastrophic, that affect us.

What might the application of this understanding look like in the managed care industry? What fundamental insight might we gain from applying fractal mathematics and an understanding of the pivotal nature of fractal transitions to analyses of health care events and costs?

**The Gutenberg-Richter Law—Managed Care’s Dilemma** Although it is unlikely that managed care decision makers would routinely encounter the name of the Gutenberg-Richter Law, the power law for CFD versus magnitude of global earthquake events, most would quickly recognize its manifestation in health care—small events are numerous, whereas large events are both rare and typically difficult to forecast or manage. The question of how to manage both healthier and sicker enrollees in a cost-effective and clinically appropriate manner has become increasingly important because of 2 clinically distinct trends.

**High Volume, Low Intensity Members.** The first trend is the expansion of chronic medication indications to increasingly healthier populations. For example, an analysis of National Health and Nutrition Education Survey IV (NHANES) data, reported by Kahn et al. in 2008, found that an estimated 78% of U.S. adults aged 20 to 80 years are candidates for at least 1 cardiovascular disease (CVD) prevention strategy—for example, providing aspirin to individuals at high CVD risk, lowering blood pressure in patients with diabetes, and reducing levels of low-density lipoprotein cholesterol—under current treatment guidelines. Applying statistical modeling to the NHANES data, Kahn et al. estimated that full implementation of all preventive strategies would reduce rates of myocardial infarction and stroke by approximately 63% and 31%, respectively, increasing life expectancy by a mean of 1.3 years for all U.S. adults. Similarly, Fletcher et al. (2009), using Markov-type modeling to estimate the impact and cost-effectiveness of primary prevention with lipid-lowering therapy according to Adult Treatment Panel III (ATP-III) guidelines, found that full ATP-III compliance would require initiating new statin therapy in 9.7 million adults and intensifying treatment in another 1.4 million adults.

The trend toward expanded indications for chronic medications appears to be increasing in intensity and scope. A November 2008 study, conducted by Ridker et al. as part of the Justification for the Use of Statins in Prevention: An Intervention Trial Evaluating Rosuvastatin (JUPITER) trial, randomly assigned 17,802 “apparently healthy” subjects without hyperlipidemia but with elevated levels of C-reactive protein to rosuvastatin 20 mg daily or placebo. After a median of 1.9 years follow-up, rates of major cardiovascular events (measured per 100 person-years of follow-up; combined outcome of myocardial infarction, stroke, arterial revascularization, hospitalization for unstable angina, or death from cardiovascular causes), were 0.77 for rosuvastatin and 1.36 for placebo (hazard ratio for rosuvastatin = 0.56, 95% CI = 0.46-0.69). In an interview conducted shortly after release of the JUPITER findings, the director of the National Heart, Lung, and Blood Institute indicated that the study would become part of the effort to “generate an evidence-based, comprehensive set of clinical guidelines for primary-care practitioners to help adult patients reduce their risk for cardiovascular disease.”
It is widely recognized that this expansion of chronic medication indications for primary prevention comes at a high economic cost, primarily because of the large and growing numbers of relatively healthy people who would be treated. Pletcher et al. found that the annual net cost of bringing all U.S. adults into full compliance with ATP-III guidelines (after accounting for medical cost savings attributable to avoidance of cardiac events) would be $3.6 billion, an estimated $42,000 per quality-adjusted life year (QALY) if low-intensity statin treatment costs $2.11 per pill.22 Similarly, Ramsey et al. found that the cost of using atorvastatin for primary prevention of cardiovascular events in patients with type 2 diabetes was $137,276 per QALY over a 5-year time horizon, $3,640 per QALY over a 10-year time horizon, and cost saving only after 25 years.18 Kahn et al. found that over a 30-year time horizon, the only cost-effective CVD prevention strategy was smoking cessation.19 These estimated costs are enough to give a decision maker pause in attempting to predict or manage the care required by these patients, especially considering that cost estimates typically do not account for the potential clinical risks associated with use of chronic medications for long-term primary prevention. For example, in commenting on the JUPITER trial, Hlatky (2008) pointed out that the lack of long-term safety data for rosuvastatin “is clearly important in considering committing low-risk subjects without clinical disease to 20 years or more of drug treatment.”19

**Low Volume, High Intensity Members.** The second trend affects the other end of the cost/intensity spectrum—the increased development and use of very high-cost medications with marginal clinical benefits in the treatment of catastrophic illnesses with low prevalence. For example, Curtiss (2006) observed that the cost of adding lenalidomide to a dexamethasone regimen for treatment of multiple myeloma was more than $6,000 per month, at a higher risk of serious side effects that included febrile neutropenia and deep vein thrombosis, in exchange for a reduction in median time to progression of disease of only 17 weeks (19.9 weeks for lenalidomide and dexamethasone vs. 37.1 weeks for dexamethasone alone).20 Schrag (2004) observed that a near-doubling of the median survival rate for metastatic colorectal cancer (MCC), from approximately 12 months in the mid-1990s to approximately 21 months a decade later, had been accompanied by a staggering 340-fold increase in drug costs21 for a typical 8-week course of chemotherapy, the cost of fluorouracil and leucovorin (the “Mayo Clinic regimen”) was $63, whereas the cost of a 4-drug regimen consisting of fluorouracil, leucovorin, oxaliplatin, and bevacizumab was $21,033. Similarly, the cost of a combination of irinotecan and cetuximab for second- and third-line treatment of MCC, producing a 1.7-fold increase in median survival, was $30,790 for an 8-week course. For all 56,000 patients with either new or recurrent MCC in the United States in 2004, Schrag estimated a total 8-week initial treatment cost of approximately $1.2 billion using therapies indicated in 2004.21 Prosser et al.’s (2004) estimates of the cost of treating newly diagnosed nonprimary progressive multiple sclerosis with interferon beta-1a, interferon beta-1b, or glatiramer acetate are even more striking; with a projected treatment duration of 10 years, interferon beta-1a was the most cost-effective treatment at $2.2 million per QALY for women and $1.8 million per QALY for men, compared with no treatment. Over a 5-year time horizon, a “no treatment” strategy yielded more estimated QALYs than did any of the drug treatment strategies.22

**Hurricanes, Large Rocks, and Medical Cost Outliers—Management Clues Provided by Studying Fractal Transitions**

The provocative question of whether to provide drug treatments of limited cost-effectiveness, although important and raised elsewhere,21 is beyond the scope of decisions made by most managed care organizations, which generally cover all drug treatments that are approved by the U.S. Food and Drug Administration. A more typical dilemma faced by a managed care decision maker is when and with which patients to intervene by providing services intended to enhance the clinical and economic benefits of treatment. Managed care organizations routinely make such decisions that affect their members—for example, whether to provide medication adherence programs to primary prevention patients, how often patients with various diseases should be contacted by a nurse telephone advice service, which patients and disease states are appropriate for step-therapy programs, or which drugs should be subject to prior authorization requirements. Fundamentally, the challenge is targeting the right program to the right patient.

The question of where to target interventions is certainly not unique to managed care. A surprisingly good analogy to managed care targeting is found at construction sites prior to groundbreaking for roads, bridges, and buildings. Civil engineers must determine how powerful a machine is required to dig foundations and whether the use of explosives is necessary. Gaussian statistical analyses cannot effectively answer these questions because an extraordinary volume of dirt and rock samples, and therefore extraordinary expense, would be required to include a sufficient number of large rocks in the sample. A few civil engineers are beginning to use fractal mathematics to address these questions.23–26

An example of the benefits of fractal analysis is shown in Figures 1 and 2. Figure 1 may initially seem familiar to managed care decision makers who routinely examine highly skewed distributions of medical cost or event data, with a classic pattern of high volume/low magnitude (left side) and low volume/high magnitude (right side) events. However, Figure 1 is actually a histogram of the sizes (in millimeters [mm]) of gravel, cobble, and boulder particles at the site of a proposed mass transit system at Sky Harbor Airport in Phoenix, Arizona, in 2003. To assess the feasibility of tunneling at the airport, it was necessary to estimate...
log transformed, the result is a linear function; hence, Figure 2 reveals a distinct linear trend that abruptly changes at a fractal transition point of approximately 30 mm (approximately 1.2 on log10 scale). The fractal transition marks a critical point in the physics of the system; this is the scale at which the soil’s “matrix,” or underlying structure, changes for small versus large rocks. This information was used by engineers both to understand better the geophysical characteristics of the site and to forecast the probability of encountering a 2-foot boulder. Using the fractal transition as a guide to the logarithmic trend expected for large rocks (i.e., extending the trend line for larger rocks from the 2-foot marker), calculations indicated that tunneling equipment would encounter a large boulder at each 100 cubic meters of material, making tunneling infeasible. This information was taken into account in making the decision not to tunnel at Sky Harbor Airport. Subsequent excavation at the site of a new control tower confirmed boulder counts consistent with the prediction based on fractal analysis.

Like typical histograms of health care utilization and cost experience in managed care, Figure 1 provides no information about the probability of encountering a catastrophic event, in this example a 2-foot boulder. The mean particle size of approximately 7 mm (0.28 inches) is clearly not informative. In contrast, in Figure 2 the CFS of particle sizes have been plotted against their magnitudes in log-log space. When power laws are
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Current Uses of Fractal Analyses in Health Care

Although the technique of plotting cumulative frequencies against magnitude in log-log space is both simple and powerful, we can find no evidence that it has been applied to analyses of medical costs or events in published work. PubMed searches on the terms power law costs logarithm, power law expenditures logarithm, Gutenberg-Richter costs logarithm, Gutenberg-Richter expenditures logarithm, fractal costs, and fractal expenditures identified only 10 studies, many of which involved the Fibrillation Registry Assessing Costs, Therapies, Adverse events and Lifestyle (FRACTAL) study, which is unrelated to fractal mathematics. Only a few managed care studies were identified, most published in the mid-1990s. Two, conducted by Davis and Lowell, used the Gutenberg-Richter power law to examine the adequacy of a behavioral health network and the relationship between fiscal structure and costs in mental health care systems.\textsuperscript{27,28} Several additional studies, conducted by business forecaster H. Richard Priesmeyer and colleagues, applied chaos theory (mathematically similar to fractal analysis) to business tasks such as evaluations of expert systems, prediction of the costs incurred by various business units within a health care system, and management of accounts receivable backlogs, and to clinical tasks such as knee arthroscopy.\textsuperscript{29-31} Although our PubMed search was not intended to be a comprehensive literature review, it reveals ample opportunity for contribution to the published managed care literature.

Fractal concepts are used much more frequently in the...
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Fractal Analyses of Medical Costs and Events—A Proposed Underlying Theory

Although the concept of applying fractal mathematics to medical events and costs in managed care appears to be new, use of this type of mathematical application in efforts to improve the understanding of human behavior is decades old. For example, Zipf posited in 1949 that the relationship between city population size and frequency follows a power law distribution. His theory spawned a huge body of research conducted by demographers and economists, almost all of it supporting his hypothesis. More recently, biologist Ethan Decker and his colleagues (2007) argued that the distribution of resources within and among cities should follow fractal branching patterns similar to those in natural systems such as river basins and biological organisms. For example, distributions of road networks arrayed in a fractal pattern “determine the rate of flow of people and goods in cities, in much the same way that the cardiovascular system determines the rate of oxygen delivery to cells.” Interestingly, Decker et al. identified (although they did not name) a fractal transition point, a point at which the log-log mathematical relationship between cumulative frequency and population size changed, signaling a change in “fundamental demographic (i.e., ecological) behaviors.”

As a starting point that is intended to be hypothesis-generating, we posit that it may be useful to conceptualize the array of health care costs paid by a managed care organization as a system of natural phenomena, in which frequent low-intensity events and rare high-intensity events contribute to the outcomes experienced by the whole system. As in any natural system, low- and high-intensity medical events will likely display fractal scaling behavior. We expect that low- and high-intensity events may be characterized well by a single power law, up to a point. At that point, the fractal transition point, the relationship may change. If the system of health care costs behaves as have other natural systems in previous scientific work, the fractal transition point may reveal something important about the physics of the system, that is, what “drives” it. Fundamental new insights into both the characteristics of the system and its appropriate management could be gained. However, as is always true of new technological approaches, the exact use of the method, and the specific insights that could be yielded from it, are impossible to determine before examining the data. An “outside-the-box” perspective is necessary to explore this possibility.

What Might a Fractal Analysis of Medical Costs and Events Show?

Although many different analyses of medical events and costs using fractal mathematics are possible, the basic approach would be the same in each: (a) construct a database of summed medical costs or events of interest, aggregated to the member level; (b) for each specific cost level, determine the cumulative frequency (number of members with costs of that level or more); (c) sort by cumulative frequency; (d) calculate log10 of both cumulative frequency and cost, and plot cost magnitude on the abscissa (x axis) and frequency on the ordinate (y axis); and (e) observe the fractal transition point(s), that is, the point(s) on the abscissa at which the slope appears to change.

If this analysis were performed on an entire population of members, we expect that at least 2 groups of members would be identified using fractal transition points. The group at the lowest cost and highest volume might consist primarily of members who use few or no medical services, perhaps also including those using chronic medications for primary prevention. The group at the highest cost and lowest volume might include patients with catastrophic illnesses and/or those using injectable biologic therapies. An intermediate cost/volume group might include patients with long-term stable chronic diseases, such as diabetes, coronary artery disease, or multiple cardiovascular disease risk factors. Decision makers could use this information in a variety of ways.

First, we posit that the fractal transition grouping method may improve the accuracy of currently available predictive indices. For example, the relationship between chronic disease score (CDS) in a baseline year and mortality and medical cost in a subsequent year is well established. Yet the CDS, like other currently used comorbidity indices that are based on administrative data, typically explains only about 15%-20% of total health care cost. Researchers who subdivide their samples into groups formed by fractal transition points may find that the mathematical relationship between CDS and outcomes is different for members in one group than in others. Mathematically accounting for that difference in statistical modeling could increase the overall predictive accuracy of the CDS and other similar comorbidity scales. Second, demographic and clinical profiles (e.g., CDS, Charlson Comorbidity Index, or Medical Outcomes Short Form scales such as the SF-36) of the various groups could help explore what, if anything, the fractal transition points show about the underlying “physics” of health care cost generation.
also intriguing is the reverse of the first analysis—the possibility of grouping members into categories using already known risk scales, then profiling members in those categories using fractal mathematics. This approach is analogous to the current application of the hurricane forecasting model, in which the data used for a given fractal analysis are limited to a sample of hurricane landfalls from a specific area of longitude and latitude (e.g., a single metropolitan area or region), and fractal modeling of that sampled area is used to make forecasts specific to that geographic location.2,5,6 As an analogous example in health care, answers to the General Self-Rated Health (GSRH) item—“In general, would you say your health is: Excellent, Very Good, Good, Fair, Poor?”—are associated with risk of subsequent hospitalization and mortality.40,41 In a meta-analysis of 22 study cohorts reported by DeSalvo et al. in 2006, the relative risks for all-cause mortality were 1.23 (95% CI = 1.09-1.39), 1.44 (95% CI = 1.21-1.71), and 1.92 (95% CI = 1.64-2.25) for those indicating good, fair, or poor health status, respectively, compared with those reporting that their health was excellent.41 However, room for improvement in prediction is clear. In DeSalvo’s (2005) analysis of data gathered from veterans, the GSRH produced a c-statistic (area under the receiver operator curve) of 0.74 for prediction of mortality and 0.63 for prediction of hospitalization, where a 0.50 indicates chance prediction and a 1.0 represents perfect prediction.40 These GSRH results were comparable to those of more complex scales currently in use by health care researchers.60

Whatever the technique used, the general idea is to apply the understanding that the fractal transition point can provide important clues to the underlying causes of low- and high-intensity events.

State of the Art—Fractal Mathematical Cost and Event Analysis in Managed Care

Does the dearth of published information about fractal analysis in health care cost forecasting and management represent insight—a wise decision to eschew an exercise in data manipulation that is too simple to provide anything new? Or is it oversight—rejection of an analytic gem that has been “hiding in plain sight” from the managed care industry? We think that the answer is closer to the latter possibility than the former, but only time, rigorous data analysis, and the willingness of creative researchers to engage in trial and error will tell.

Seemingly disparate fields of study often have much to learn from each other. Decker et al. have observed that “common mathematical distributions may result from very general processes in natural, economic and engineered systems.”36 The similarity of findings in fractal-based research conducted in multiple fields of study may be showing us that the basic laws underlying diverse systems—biological, demographic, economic, and geological—are more similar than is generally understood.

Perhaps there is an opportunity for managed care to increase its use of an analytic approach that has been applied successfully by demographers, economists, and physical scientists to improve their understanding of the behavior of natural physical and human ecological systems. The mathematical tools that are commonly used in managed care to analyze health care costs, such as per member per month measures and even cost per QALY, may be inadequate to explain or manage care patterns that “drive” expenditures for clinical trends at dramatically different end points of the volume/intensity spectrum. Fractal mathematics may provide new and fascinating insights into care management. Our hope is that this editorial will encourage readers to explore that exciting possibility.

REFERENCES

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Fractal Mathematics in Managed Care? How a Simple and Revealing Analysis Could Improve the Forecasting and Management of Medical Costs and Events


